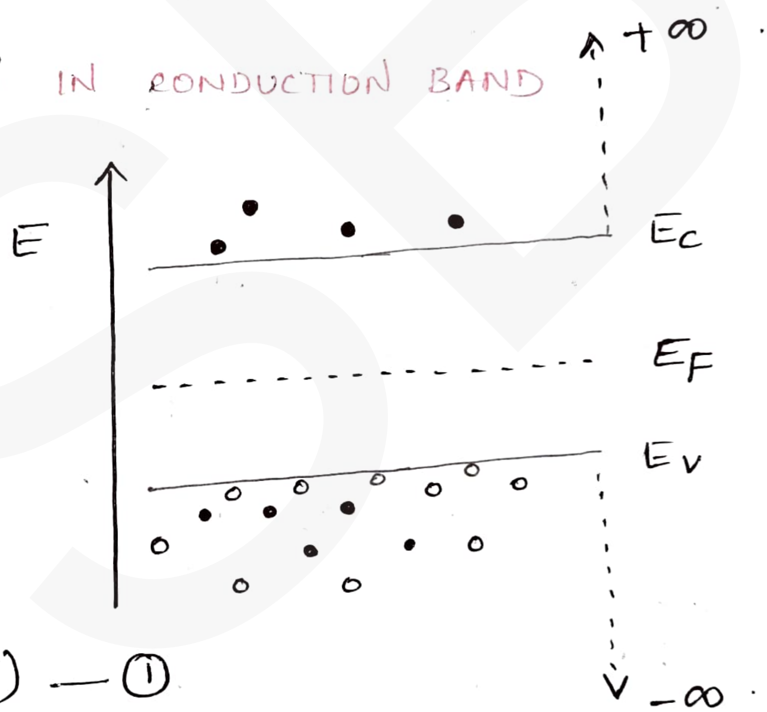
CARRIER CONCENTRATION IN INTRINSIC SEMICONDUCTORS

n : electron concentration, the number of $\bar{e}$s in conduction band per unit volume of the material is called as electron concentration.

p: hole concentration, the number of holes in valence band per unit volume of the material is called hole concentration.

DENSITY OF ELECTRONS IN CONDUCTION BAND

The no of $\bar{e}$s per unit volume in CB for energy range E to E+dE is dn



$$dn = g(E)dE\, f(E) \quad \text{—} ①$$

where,

$g(E)\,dE$ = density of states in energy range E to E+dE

$f(E)$ = probability of $\bar{e}$ occupancy according to Fermi - Dirac distribution.

conduction band ranges from E_c to $+\infty$.

$$\therefore \quad n = \int dn = \int_{E_c}^{\infty} g(E)\,dE \; f(E) \quad \text{—} (2)$$

where, E_c corresponds to bottom of CB.

∞ " " uppermost level of CB.

Density of states between E to $E+dE$ is given by,

$$g(E)\,dE = \frac{\pi}{2}\left(\frac{8m_e^*}{h^2}\right)^{3/2} E^{1/2}\,dE \quad \text{—} (3)$$

E represents energy values an $\bar{e}$ can have in CB. $\therefore$ for CB we can write,

$$E = E - E_c$$

$$\therefore g(E)\,dE = \frac{\pi}{2}\left(\frac{8m_e^*}{h^2}\right)^{3/2} (E-E_c)^{1/2}\,dE \quad \text{—} (4)$$

here,

$E - E_c$ is kinetic energy of conduction $\bar{e}$.

m_e^* is effective mass of $\bar{e}$ in CB.

h is Planck's constant.

wkt ~~Now~~, Fermi distribution function is given by

$$f(E) = \frac{1}{e^{(E-E_F)/k_BT} + 1} \quad \text{—} (5)$$

k_B is Boltzmann constant.

In CB,

$$E - E_F \gg k_B T$$

$$\therefore \quad e^{(E-E_F)/k_BT} + 1 \sim e^{(E-E_F)/k_BT}$$

$$\therefore \quad f(E) = e^{-(E-E_F)/k_BT} \quad \text{——} (6)$$

substituting (4) & (6) in (2) we get

$$n = \int_{E_c}^{\infty} \frac{\pi}{2}\left(\frac{8m_e^*}{h^2}\right)^{3/2} (E-E_c)^{1/2} \, e^{-(E-E_F)/k_BT} \, dE .$$

$$\text{or} \quad n = \int_{E_c}^{\infty} \frac{\pi}{2}\left(\frac{8m_e^*}{h^2}\right)^{3/2} (E-E_c)^{1/2} \, e^{-E/k_BT} \cdot e^{+E_F/k_BT} \, dE .$$

$$\text{or} \quad n = \frac{\pi}{2}\left(\frac{8m_e^*}{h^2}\right)^{3/2} e^{E_F/k_BT} \int_{E_c}^{\infty} (E-E_c)^{1/2} \cdot e^{-E/k_BT} \, dE$$

consider, $E - E_c = x$. & $E = x + E_c$

$dE = dx$

limits, when $E = E_c$,

$x = 0$

when $E = \infty$

$x = \infty$

$$\therefore \quad n = \frac{\pi}{2}\left(\frac{8m_e^*}{h^2}\right)^{3/2} e^{E_F/k_BT} \int_{0}^{\infty} x^{1/2} \cdot e^{-(x+E_c)/k_BT} \, dx .$$

$$\text{or} \quad n = \frac{\pi}{2}\left(\frac{8m_e^*}{h^2}\right)^{3/2} e^{E_F/k_BT} \cdot \int_{0}^{\infty} x^{1/2} \, e^{-x/k_BT} \cdot e^{-E_c/k_BT} \, dx .$$

or $$n = \frac{\pi}{2}\left(\frac{8m_e^*}{h^2}\right)^{3/2} e^{(E_F - E_c)/k_BT} \int_0^\infty x^{1/2} \cdot e^{-x/k_BT}\, dx$$

From mathematical identity :
Gamma function

$$\int_0^\infty x^{n-1} e^{-ax}\, dx = \frac{\Gamma n}{a^n}$$ we get

$$\Gamma 3/2 = \tfrac{1}{2}\Gamma 1/2 = \tfrac{1}{2}\sqrt{\pi}$$

$$\int_0^\infty x^{1/2} e^{-x/k_BT}\, dx = \frac{\Gamma 3/2}{(1/k_BT)^{3/2}} = \frac{\sqrt{\pi}}{2}(k_BT)^{3/2}$$

$$\therefore\ n = \frac{\pi}{2}\left(\frac{8m_e^*}{h^2}\right)^{3/2} e^{(E_F - E_c)/k_BT} \cdot \frac{\sqrt{\pi}}{2}(k_BT)^{3/2}.$$

or $$n = \frac{1}{4}\left(\frac{8\pi k_BT\, m_e^*}{h^2}\right)^{3/2} e^{(E_F - E_c)/k_BT}$$

or $$\boxed{n = 2\left(\frac{2\pi k_BT\, m_e^*}{h^2}\right)^{3/2} e^{(E_F - E_c)/k_BT}}$$

NOTE

gamma n is: $\Gamma n = n-1\,\Gamma n-1$

eg $\Gamma 3/2 = \frac{1}{2}\Gamma 1/2$

$\Gamma 5/2 = \frac{3}{2}\Gamma 3/2 = \frac{3}{2} \times \frac{1}{2}\Gamma 1/2.$

& identity $\Gamma 1/2 = \sqrt{\pi}$

$\therefore\ \Gamma 3/2 = \frac{\sqrt{\pi}}{2}, \quad \Gamma 5/2 = \frac{3}{4}\sqrt{\pi}$

so on. . . .