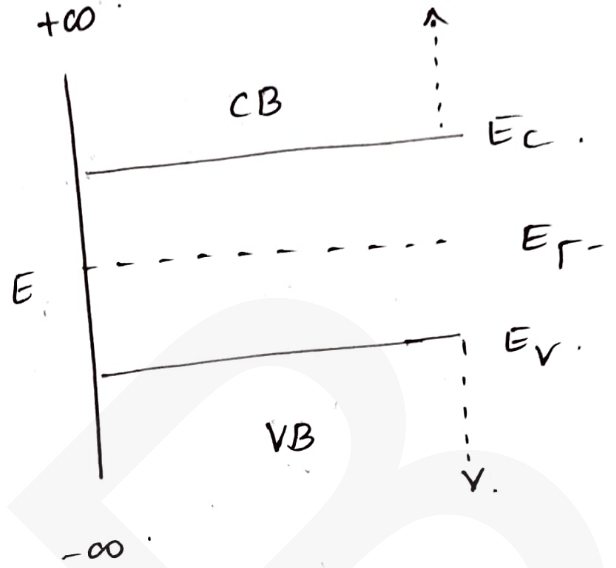
Density of holes in VB of intrinsic semiconductor



Let dp be the number of holes per unit volume in VB between the energy E & E+dE

$$dp = g(E)dE \cdot f_h(E) \quad \text{--- (1)}$$

$g(E)dE$ is density of states in energy range E & E+dE.

$f_h(E)$ is the probability of hole occupancy.

No of holes in the entire VB ranging from $-\infty$ to E_v is given by,

$$p = \int dp = \int_{-\infty}^{E_v} g(E)dE \cdot f_h(E) \quad \text{--- (2)}$$

$$g(E)dE = \frac{\pi}{2}\left(\frac{8m_p^*}{h^2}\right)^{3/2} E^{1/2}\, dE$$

For VB, E can be replaced by $(E_v - E)$, the energy level available for hole to occupy.

$(E_v - E)$ signifies kinetic energy of hole.

m_p^* is the effective mass of hole.

$$\therefore\ g(E)\,dE = \frac{\pi}{2}\left(\frac{8m_p^*}{h^2}\right)^{3/2}(E_v - E)^{1/2}\,dE \quad \text{--- (3)}$$

$f_h(E) = 1 - f(E)$
↳ probability of $\bar{e}$ occupancy.

As the presence of hole is nothing but absence of $\bar{e}$, we can write probability of hole occupancy is nothing but 1 - probability of $\bar{e}$ occupancy.

$$\therefore\ f_h(E) = 1 - \frac{1}{1 + e^{(E-E_F)/k_BT}}$$

$$\text{or } f_h(E) = \frac{\cancel{1} + e^{(E-E_F)/k_BT} - \cancel{1}}{1 + e^{(E-E_F)/k_BT}}$$

here $E - E_F$ is negative quantity

$$\therefore\ e^{(E-E_F)/k_BT} \ll 1 \text{ and}$$

$$1 + e^{(E-E_F)/k_BT} \sim 1$$

$$\therefore\ f_h(E) = e^{(E-E_F)/k_BT} \quad \text{--- (4)}.$$

substituting (3) & (4) in (2) we get,

$$p = \int_{-\infty}^{E_V} \frac{\pi}{2}\left(\frac{8m_p^*}{h^2}\right)^{3/2}(E_V - E)^{1/2}\, e^{(E-E_F)/k_BT}\,dE.$$

$$p = \frac{\pi}{2}\left(\frac{8m_p^*}{h^2}\right)^{3/2} e^{-E_F/k_BT} \int_{-\infty}^{E_V} (E_V - E)^{1/2}\, e^{E/k_BT}\, dE$$

consider $E_V - E = x$ or $E = E_V - x$

$-dE = +dx$ or $dE = -dx$.

limits when $E = -\infty$, $x = \infty$

when $E = E_V$, $x = 0$

$$\therefore p = \frac{\pi}{2}\left(\frac{8m_p^*}{h^2}\right)^{3/2} e^{-E_F/k_BT} \int_{+\infty}^{0} x^{1/2}\, e^{(E_V - x)/k_BT}\, dE$$

$$\text{or } p = \frac{\pi}{2}\left(\frac{8m_p^*}{h^2}\right)^{3/2} e^{-E_F/k_BT} \int_{\infty}^{0} x^{1/2}\, e^{E_V/k_BT} \cdot e^{-x/k_BT} (-dx)$$

$$\text{or } p = \frac{\pi}{2}\left(\frac{8m_p^*}{h^2}\right)^{3/2} e^{(E_V - E_F)/k_BT} \int_{\infty}^{0} x^{1/2}\, e^{-x/k_BT} (-dx).$$

$$\int_{\infty}^{0} (-dx) = \int_{0}^{\infty} dx.$$

$$\text{or } p = \frac{\pi}{2}\left(\frac{8m_p^*}{h^2}\right)^{3/2} e^{(E_V - E_F)/k_BT} \int_{0}^{\infty} x^{1/2}\, e^{-x/k_BT}\, dx.$$

Using Gamma function

$$\int_0^\infty x^{1/2}\, e^{-x/k_BT} = \frac{\sqrt{\pi}}{2} (k_BT)^{3/2}$$

$$\therefore \quad p = \frac{\pi}{2}\left(\frac{8m_p^*}{h^2}\right)^{3/2} e^{(E_V - E_F)/k_B T} \times \frac{\sqrt{\pi}}{2}(k_B T)^{3/2}$$

$$\text{or} \quad p = \frac{1}{4}\left(\frac{8\pi m_p^* k_B T}{h^2}\right)^{3/2} e^{(E_V - E_F)/k_B T}$$

$$\boxed{p = 2\left(\frac{2\pi m_p^* k_B T}{h^2}\right)^{3/2} e^{(E_V - E_F)/k_B T}}$$