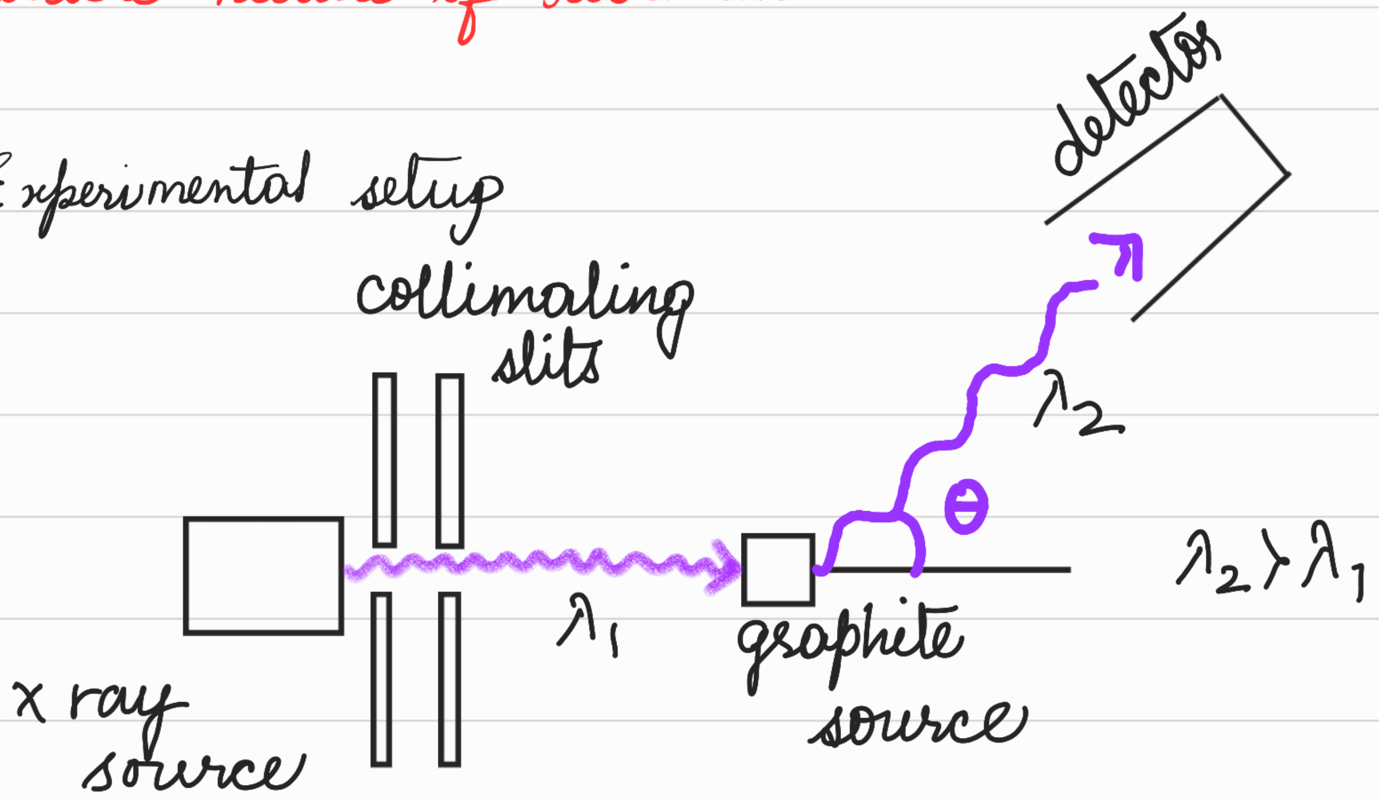
Compton Effect

Definition

The compton effect refers to the increase in the wavelength of high energy EM radiation (x or γ rays) when they are scattered by loosely bound or free electrons in a material, demonstrating the particle nature of radiation.

Experimental setup



→ A monochromatic beam of x rays is produced using an x-ray tube.

→ The beam is directed to target with loosely bound $\bar{e}$ which acts as scattering material.

→ x rays are scattered by e^{-s} in target material.

→ The scattered x rays are observed at different scattering angle(θ) using spectrometer.

→ The wavelength and intensity of scattered x rays are recorded using a detector.

Physical interpretation

→ In this process an incident photon collides with an $\bar{e}$ in the material.

→ During the interaction, the photon transfers part of its energy & momentum to the $\bar{e}$.

→ The $\bar{e}$ recoils with kinetic energy

→ The scattered photon therefore has lower energy and a longer wavelength.

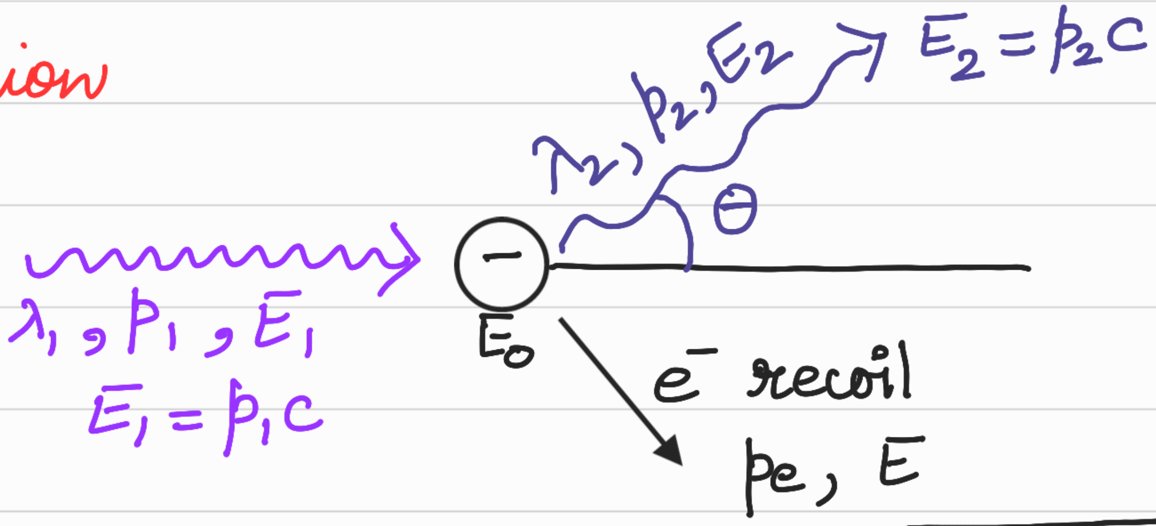
Assumptions :

→ The $\bar{e}$ is considered free or loosely bound

→ The $\bar{e}$ is initially at rest.

→ The collision occurs between one photon & one $\bar{e}$. (two body interaction)

Derivation



$$E = \sqrt{E_0^2 + p_e^2 c^2}$$

$$E_0 = m_0 c^2$$

Compton shift $\Delta\lambda = \lambda_2 - \lambda_1$

In Compton effect

Total relativistic energy is conserved.

Total momentum is conserved.

Momentum conservation

$$\vec{p_1} + 0 = \vec{p_2} + \vec{p_e}$$

or $$(\vec{p_1} - \vec{p_2}) = \vec{p_e}$$

or $$p_1^2 + p_2^2 - 2p_1 p_2 \cos\theta = p_e^2 \quad \text{—— (1)}$$

Energy conservation

$$E_1 + E_0 = E_2 + E$$

or $$p_1 c + m_0 c^2 = p_2 c + \sqrt{m_0^2 c^4 + p_e^2 c^2}$$

or $$p_1c - p_2c + m_0c^2 = \sqrt{m_0^2c^4 + p_e^2c^2}$$

or $$c(p_1 - p_2) + m_0c^2 = \sqrt{m_0^2c^4 + p_e^2c^2}$$

squaring on both sides

$$\left.\begin{aligned} c^2(p_1-p_2)^2 + \cancel{m_0^2c^4} \\ + 2c(p_1-p_2)m_0c^2 \end{aligned}\right\} = \cancel{m_0^2c^4} + p_e^2c^2$$

substituting for p_e from ① we get

$$\left.\begin{aligned} \cancel{c^2p_1^2} + \cancel{c^2p_2^2} - 2c^2p_1p_2 \\ + 2c(p_1-p_2)m_0c^2 \end{aligned}\right\} = \begin{aligned} \cancel{c^2p_1^2} + \cancel{c^2p_2^2} \\ - 2c^2p_1p_2\cos\theta \end{aligned}$$

or $$2c(p_1-p_2)m_0c^2 = -2c^2p_1p_2\cos\theta + 2c^2p_1p_2$$

or $$\cancel{2}c(p_1-p_2)m_0\cancel{c^2} = \cancel{2}\cancel{c^2}p_1p_2(1-\cos\theta)$$

or $$m_0c\left(\frac{h}{\lambda_1} - \frac{h}{\lambda_2}\right) = \frac{h^2}{\lambda_1\lambda_2}(1-\cos\theta)$$

or $$m_0c\left(\frac{\lambda_2-\lambda_1}{\cancel{\lambda_1\lambda_2}}\right)\cancel{h} = \frac{h^{\cancel{2}}}{\cancel{\lambda_1\lambda_2}}(1-\cos\theta)$$

or $$\lambda_2 - \lambda_1 = \frac{h}{m_0c}(1-\cos\theta)$$

or $$\boxed{\Delta\lambda = \frac{h}{m_0c}(1-\cos\theta)}$$